RG

Stratified Turbulence

7/31/2019

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Review of unstratified b. l. turbulence

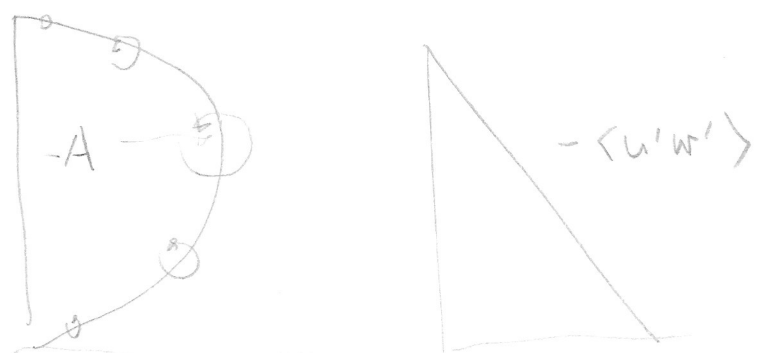
Key empirical findings:

1) Quadratic drag

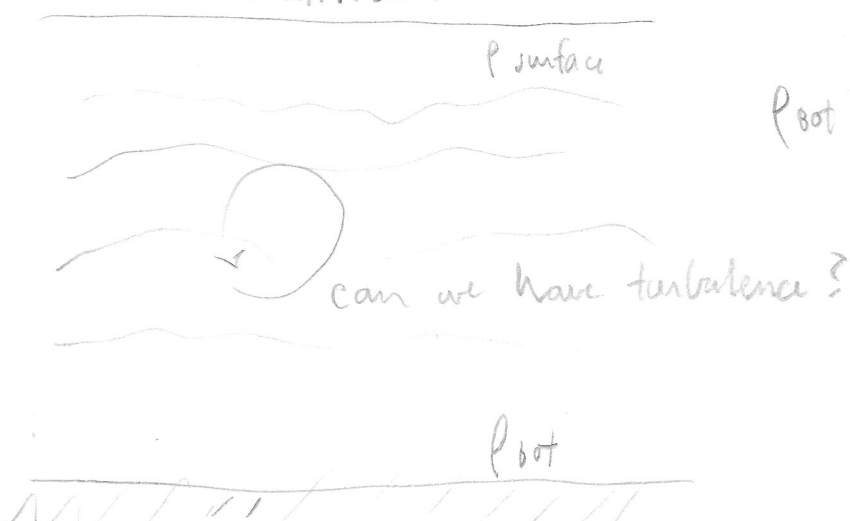
2) Log Layer

3) Eddy Viscosity $A = u_{TURB} L_{TURB}$

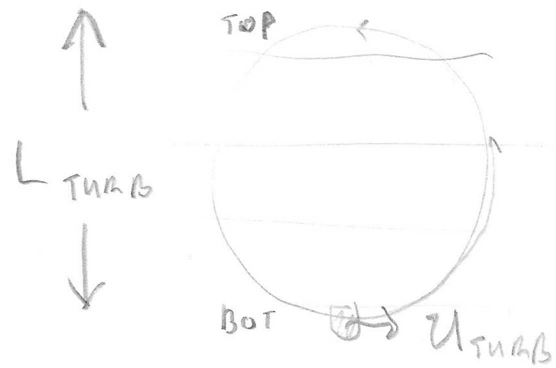
$$u_{TURB} = |\langle u'w' \rangle|^{\frac{1}{2}}, \quad L_{TURB} \approx \kappa z \left(1 - \frac{z}{h}\right)^{\frac{1}{2}}$$



Now with stratification



$\rho_{bot} > \rho_{surface}$



if the parcel doesn't have enough energy to turn over then it is an internal wave

$KE: \frac{1}{2} u_{TURB}^2$, Bernoulli: $\frac{1}{2} u^2 + p' = B$

change of PE $= \frac{1}{2} \Delta\rho\, g\, L_{TURB}$ $\qquad \Delta\rho = \rho_{Bot} - \rho_{Top}$

$$\cancel{\tfrac{1}{2}} \bar{\rho}\, u_{TURB}^2 = \cancel{\tfrac{1}{2}} \Delta\rho\, g\, L_{TURB}$$

substitute in: $N^2 = -\frac{g}{\rho}\frac{\partial\rho}{\partial z}$ fastest frequency^2 of internal waves

$\Rightarrow \quad u_{TURB}^2 = N^2 L_{TURB}^2$

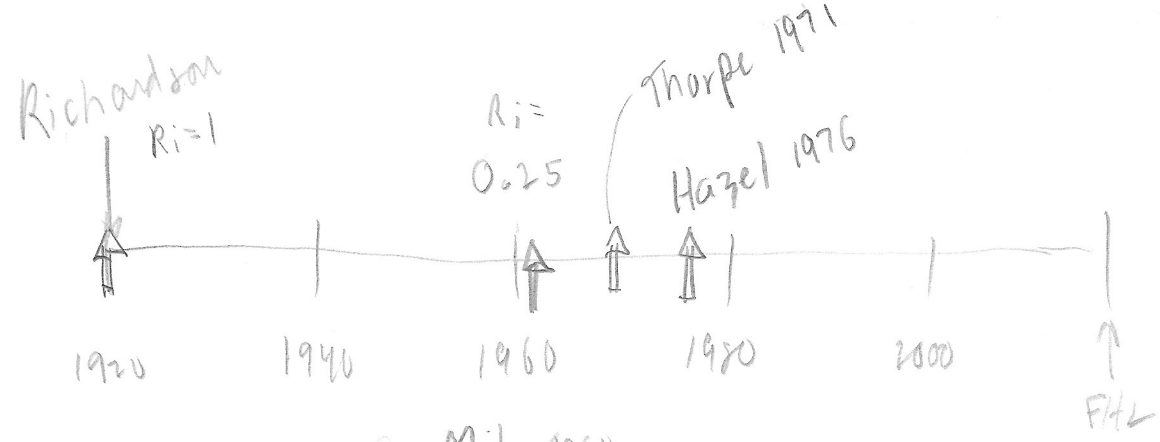
or $\boxed{L_{TURB} \leq u_{TURB}/N}$ if we satisfy the inequality then the wheel can turn over and the flow is turbulent

or $$\frac{N^2}{u_{TURB}^2 / L_{TURB}^2} < 1$$

$\{$ ~~or~~ shear^2 of turbulence: assume it is proportional to the square of the mean shear

$$\Rightarrow \frac{N^2}{\left(\frac{\partial \langle u \rangle}{\partial z}\right)^2} < 1 \equiv \text{Richardson \#},\ Ri$$

really gradient Richardson #

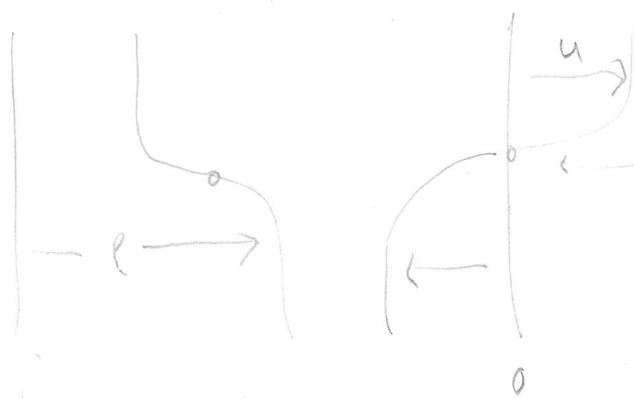


Miles (1961)
Howard (1961)

$Ri > 0.25$ flow is stable

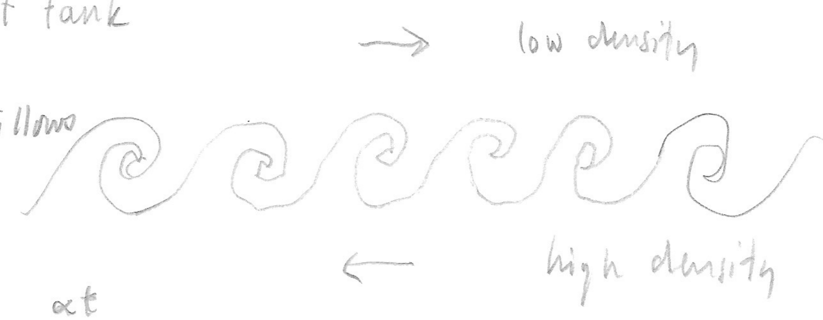
$Ri < 0.25$ Necessary but not sufficient condition for flow to be unstable

Hazel



if min Ri is at the inflection point then $Ri = \frac{1}{4}$ is sufficient.

Thorpe Experiments : tilt tank



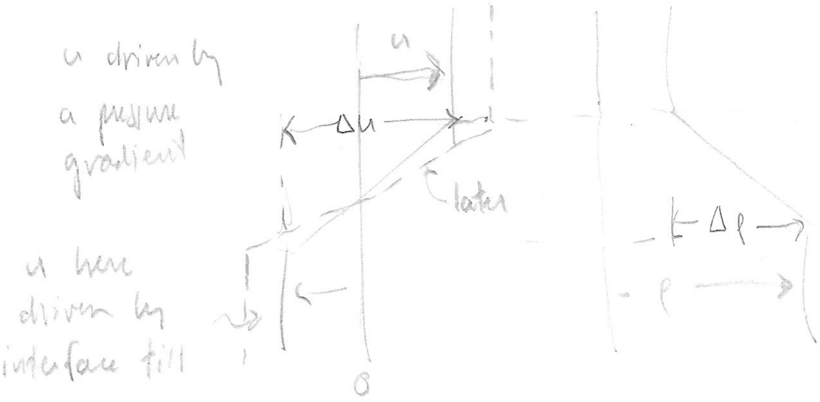
growth rate $a = a_0 e^{\alpha t}$

$$\alpha = \frac{\partial u_0}{\partial z} \, 0.2 \, (1 - 4 Ri)$$

$\Rightarrow 0$ at $Ri = 1/4$

Marginally Critical Stratified Turbulence

Thought experiment: forced shear layer: <u>u is accelerating</u>

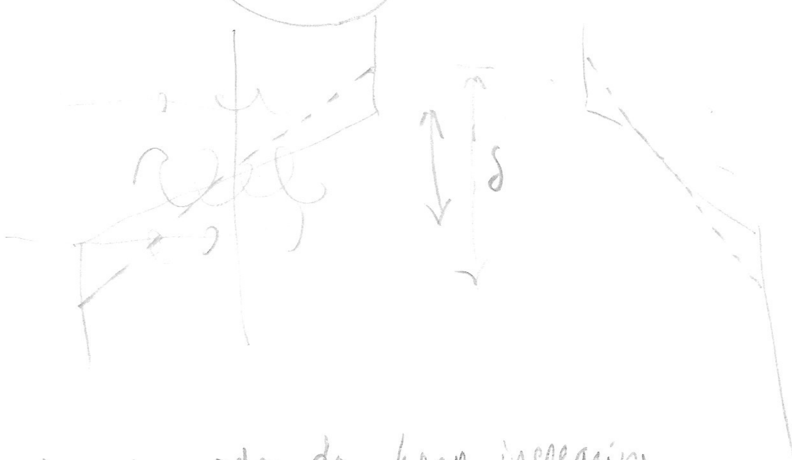


Imagine $Ri > \frac{1}{4}$

Then shear will increase while stratification is unchanged

when Ri gets to $\frac{1}{4}$ there will be turbulent mixing

layer entrains + gets <u>thicker</u>



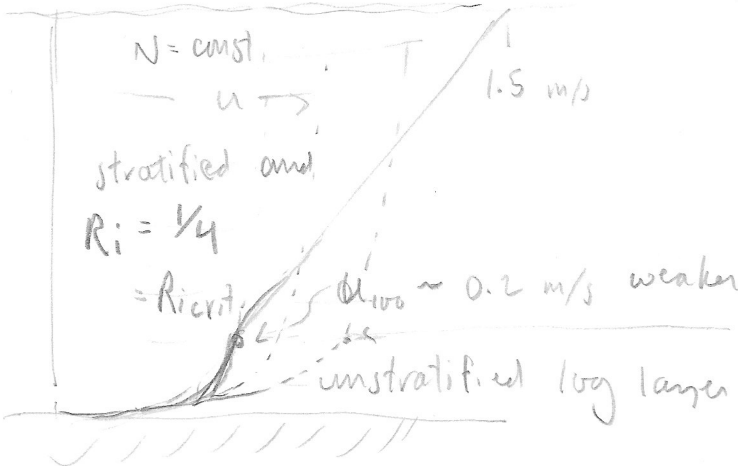
$$\boxed{\frac{\Delta\rho\, g\, \delta}{(\Delta u)^2} = 0.25} \quad (*)$$

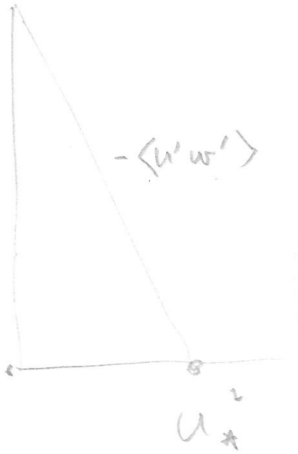
or $$\frac{\dfrac{\Delta\rho\, g}{\rho_0\, \delta}}{(\Delta u/\delta)^2}$$

(*) ⇒ in order to keep increasing Δu while maintaining $R_i = 1/4$ <u>Requires</u> that δ get thicker.

Entrainment means advection of fluid from a region without mixing into a region with mixing.

pressure gradient driving the flow





$|\langle u'w' \rangle|^{\frac{1}{2}}$

$$A = L_{TURB}\, U_{TURB} = \frac{-\langle u'w' \rangle}{\partial\langle u \rangle / \partial z} = \frac{U_{TURB}^2}{Ri_{crit}^{-1/2}\, N}$$

$$Ri_{crit} = \frac{N^2}{(\partial u/\partial z)^2} = .25$$

$$\Rightarrow L_{TURB} = Ri_{crit}^{1/2} \frac{U_{turb}}{N} = 0.5 \frac{U_{TURB}}{N}$$

$$\text{or} \quad L_{TURB} = 0.5 \frac{u_*}{N}$$

This is close to the Ozmidov scale

$$L_O = \left(\frac{\varepsilon}{N^3}\right)^{\frac{1}{2}}$$

Just need to estimate ε (dissipation) for the b. l. case above

Invoke : Production of TKE $\cong \varepsilon + \frac{g}{\rho}\langle \rho' w' \rangle$ ← Buoyancy Flux

$\frac{\text{Buoy}}{\varepsilon} = \Gamma$ called "efficiency" ~ 0.2

$\frac{\text{Buoy}}{\text{Prod}} = R_f$ really is efficiency $\simeq 0.167$

$$\text{Prod} = -\langle u'w' \rangle \frac{\partial u}{\partial z}$$

$$\varepsilon = (1 - R_f) \langle u'w' \rangle \frac{\partial u}{\partial z}$$

$$\Rightarrow \quad \frac{\varepsilon}{N^3} = (1 - R_f)\, R_i^{-1/2} \frac{U_{TURB}^2}{N^2}$$

$$L_O = \left(\frac{\varepsilon}{N^3}\right)^{1/2} = (1 - R_f)^{1/2}\, R_i^{-1/4} \frac{U_{TURB}}{N}$$

similar to the overturn scale (Thorpe scale)

of turbulence